**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2018**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 1 of term 4, 2018**

**Question 8 (5 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states the correct critical values * writes correct equations for each sub-interval * determines correct conclusion for * correct solution for * provides the correct solution for | 1  1  1  1  1 |

**Question 9 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| If    then | |
| Mathematical behaviours | Marks |
| * combines the two fractions * inverts the expression and multiplies by the complex conjugate * deduces the correct real and imaginary parts | 1  1  1 |

**Question 9(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| From calculator | |
| Mathematical behaviours | Marks |
| * states the value required quoting the answer to eight places | 1 |

**Question 9(c) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Since  if  and  .  The smallest possible solution is clearly  . | |
| Mathematical behaviours | Marks |
| * solves for the required  giving the answer in radians | 1 |

**Question 9(d) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| From (b) we deduce that . | |
| Mathematical behaviours | Marks |
| * combines the results of parts (a) and (b) to infer the exact value | 1 |

**Question 10(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Given that is a solution of  then  Real parts give that  and imaginary parts imply that | |
| Mathematical behaviours | Marks |
| * substitutes the value  into the equation * deduces the required values of  and | 1  1 |

**Question 10(b) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since  is a solution of the polynomial equation then so is  Hence the quartic has a factor  By long division    By the quadratic formula the equation  has solutions  Hence the required three other solutions are  and | |
| Mathematical behaviours | Marks |
| * correctly notes that the conjugate is also a solution * notes that the quartic has a factor * factorises the quartic * deduces the required extra solutions of the quartic | 1  1  1  1 |

**Question 11(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * evaluates the cross product correctly | 1 |

**Question 11(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| and 3 are normal to the planes and  and is perpendicular toboth of these normals. (\*)  So is parallel to vectors in each of the planes and  So is parallel to the line of intersection of these planes, i.e. L. (\*\*) | |
| Mathematical behaviours | Marks |
| * obtains result (\*) * completes the argument correctly (\*\*) | 1  1 |

**Question 11(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since is parallel to L, a vector equation for L has the form  As lies in L, we may assume that    So is a vector equation for L. | |
| Mathematical behaviours | Marks |
| * evaluates * obtains a correct vector form of equation | 1  1 |

**Question 11(d)(i) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| The equations reduce to (\*)  The planes , and have infinitely points in common if the third equation reduces to , i.e. if and | |
| Mathematical behaviours | Marks |
| * attempts simultaneous reduction of the equations for , and * derives equation (\*) * deduces the correct solutions | 1  1  1 |

**Question 11(d)(ii) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| The planes , and have no point in common if equations (\*) in d(i) are inconsistent,  i.e. the third equation reduces to , i.e. if and | |
| Mathematical behaviours | Marks |
| * deduces the correct solution | 1 |

**Question 12 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * derives correct expression for * derives correct expression for | 1  1 |

**Question 12 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct domain * states correct range | 1  1 |

**Question 12 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct domain including mention of the * states correct range | 1+1  1 |

**Question 13(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| For  the curve lies above the line so | |
| Mathematical behaviours | Marks |
| * gives a sketch of the form of the area * writes down the correct expression for the required area * evaluates the two integrals to deduce the necessary result | 1  1  1 |

**Question 13(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Required volume of rotation is given by | |
| Mathematical behaviours | Marks |
| * writes down the appropriate expression for the volume for the two parts * deduces the correct volume | 1  1 |

**Question 14 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since , then motion is SHM with period =  . | |
| Mathematical behaviours | Marks |
| * differentiates to obtain experession for acceleration * shows motion is simple harmonic with correct period | 1  1 |

**Question 14 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct amplitude * states correct displacement equation | 1  1 |

**Question 14 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct maximum speed * states correct minimum speed | 1  1 |

**Question 14 (d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * expresses velocity as a function of time * integrates using correct upper and lower values * calculates distance travelled | 1  1  1 |

**Question 14 (e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * calculates  and * draws correct conclusion | 1  1 |

**Question 15 (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| If we write  Equating the coefficients gives  and  Hence | |
| Mathematical behaviours | Marks |
| * writes down the appropriate form of the partial fractions * compares coefficients to deduce the constants  and * integrates correctly * deduces the required result | 1  1  1  1 |

**Question 16 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| The parabola joins the orgin with the point (9,6)  Line cuts parabola within the domain if . | |
| Mathematical behaviours | Marks |
| * determines correctly the required inequality | 1 |

**Question 16 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| There is no need to compute the point of intersection. The two areas are equal if the area under the parabola over matches the area under the line.  Now  and  Thus | |
| Mathematical behaviours | Marks |
| * realises there is no need to determine the point of intersection * evaluates the areas under the line and parabola over * deduces the requisite value of | 1  1  1 |

**Question 17(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| (\*) | |
| Mathematical behaviours | Marks |
| * derives the correct Z-limits * obtains the correct answer | 1  1 |

**Question 17(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| From the calculator (or tables)  That is, the upper quartile of the distribution is 6.255 | |
| Mathematical behaviours | Marks |
| * calculates 0.675 as the upper quartile in the normal distribution * obtains the correct answer | 1  1 |

**Question 17(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| From the formula sheet (\*)  Since , and  and so  So the sample size needs to be at least 163 | |
| Mathematical behaviours | Marks |
| * uses the formula (\*) * obtains the correct answer | 1  1 |

**Question 17(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| , and  So the 95% confidence interval is  i.e. 4 | |
| Mathematical behaviours | Marks |
| * evaluates E correctly * dervies the correct limits for the confidence interval | 1  1 |

**Question 17(e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The statement is correct.  This is because 5.04, the mean number of migraine attacks for untreated people, is much greater than 4.74, the upper limit of the upper limit of the 95% confidence interval for the average number of migraine attacks for people given acupuncture. | |
| Mathematical behaviours | Marks |
| * accepts the claim * gives a valid reason based on the confidence interval | 1  1 |

**Question 17(f) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| We determine a 95% confidence interval for the average number of migraine attacks for people who take the drug.  , and  So the 95% confidence interval is  i.e.    These tests DO NOT clearly show that the drug is more effective than acupuncture in treating migraine attacks, (\*) because 4 and the confidence intervals for and overlap considerably. | |
| Mathematical behaviours | Marks |
| * constructs a confidence interval for * sets the confidence level at least as high as 90% * obtains correct limits for the confidence interval for * obtains the correct conclusion about the tests (\*) * gives a valid reason in terms of overlapping confidence intervals | 1  1  1  1  1 |

**Question 18(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| and , (\*) because  when or  So rocket A returns to the horizontal plane H when  ,  andthe distance of this point from O is | |
| Mathematical behaviours | Marks |
| * obtains expression for (\*) * obtains expression for (\*) * solves for t * obtains the correct answer | 1  1  1  1 |

**Question 18(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since is constant, (\*)  If the flight paths meet, for some values of and  So and (\*\*)  If and  So and hence . | |
| Mathematical behaviours | Marks |
| * solves for (\*) * locates points where the flight paths meet * obtains correct answer for | 1  1  1 |

**Question 18(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| From part (b), if the rockets collide (\*)  Also from (b), and so that implying that  So the speed of rocket B is (approximately) if the rockets collide. | |
| Mathematical behaviours | Marks |
| * uses (\*) * obtains correct answer | 1  1 |

**Question 19 (7 marks)**

|  |  |
| --- | --- |
| Solution | |
| If the cross section of the sphere is given by  then the volume of water is given  by      Now the volume of the complete sphere is    so the computed volume is a fraction    When we put    the fraction reduces to ½ as anticipated as this denotes the sphere is half full.  When    the fraction equals 1 corresponding to a completely full sphere | |
| Mathematical behaviours | Marks |
| * writes down an appropriate integral….. * …with the correct limits * integrates correctly and inserts the limits * simplifies the expression * evaluates the correct fraction of the complete volume * interprets correctly the case when * interprets correctly the case when | 1  1  1  1  1  1  1 |

**Question 20 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * rewrites integral in terms of * determines correct antiderivative * rewrites antiderivative in terms of | 1  1  1 |

**Question 20(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * differentiates implicitly to determine * establishes | 1  1 |

**Question 20 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * differentiates implicitly to determine * establishes | 1  1 |

**Question 20 (d) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| The second model predicts a slightly smaller population. | |
| Mathematical behaviours | Marks |
| * correct population equation using logistic equation * calculates population size * correct population equation using second model * calculates population size * compares predicted population sizes | 1  1  1  1  1 |